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Tests of the naturalness of the coupling constants in ChPT at order p^6

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Abstract. We derive constraints on combinations of $O(p^6)$ chiral coupling constants by matching a recent two-loop calculation of the πK scattering amplitude with a set of sum rules. We examine the validity of the natural expectation that the values of the chiral couplings can be associated with physics properties of the light resonance sector. We focus, in particular, on flavor symmetry breaking of vector resonances. A resonance chiral Lagrangian is constructed which incorporates flavor symmetry breaking more completely than was done before. We use πK unsubtracted sum rules as tests of the modelling of the resonance contributions to the chiral couplings. In some cases the $O(p^6)$ couplings are found not to be dominated by the resonance contributions.

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1 Introduction

Important progress in the description of QCD via effective theories was achieved by the extension of the chiral expansion formalism [1–3] to the order p^6 [4–6]. This raises the hope of attaining high precisions in the description of low energy physics using the chiral expansion, even in the case of the three-flavor expansion which is expected to converge more slowly than the two-flavor one. A large number of quantities have already been computed at chiral order six starting from the work of [7]. Some representative examples concerning the two-flavor case are in [8, 9] and in the three-flavor case in [10–15].

In practice, including the $O(p^6)$ corrections was shown to clearly bring significant improvement for the two-flavor expansion [7, 8]. In this case, the corrections are dominated by the chiral logarithms, the coefficients of which are known in terms of the $O(p^2)$ and $O(p^4)$ coupling constants [1], while the corrections proportional to the $O(p^6)$ couplings are comparatively smaller. The situation for the three-flavor expansion is different, in that the role of the $O(p^6)$ couplings is much more important. As an example, in order to determine the CKM matrix element V_{us} at the one percent level based on experimental data on $K \to \pi l \nu$ decays it is necessary to know the values of the two low energy constants (LECs) $C^{\rm r}_{12}$ and $C^{\rm r}_{34}$ (see e.g. [16]).

In so far as only the order of magnitude of the chiral LECs is concerned, it is possible to make very simple and general statements [17, 18]. The order of magnitude can be argued to depend only on F_π and on the chiral scale $\Lambda_\chi \simeq M_\rho$, so that the typical size of the $O(p^4)$ LECs should be $L^{\rm r}_i \sim F^2_\pi/M^2_\rho$ and that of the $O(p^6)$ LECs should be $C^{\rm r}_i \sim F^2_\pi/M^4_\rho$. The natural question which arises, then, is whether it is possible to make more quantitative estimates relating the values of the LECs to known properties of the light resonances in the QCD spectrum. A detailed study along this line was performed in [19] in which it was observed that it is indeed possible to reproduce the values of the $O(p^4)$ LECs $L^{\rm r}_i(\mu)$ with $\mu = M_\rho$, which had previously been determined in a model independent way [3], in terms of observables from the light resonance sector.

A justification for such a relationship is provided by the chiral sum rules (see e.g. [2] for a list). A typical example, which was analyzed in [20, 21] is the LEC $L^{\rm r}_{10}$ which can be expressed as a convergent integral in terms of spectral functions which can be determined experimentally from τ decays. To a good approximation, the integral is found to be saturated by the contributions from the $\rho(770)$ and $a_1(1230)$ mesons. In more complicated situations, for which the integrands cannot easily be measured, one can appeal to the large N_c expansion. Indeed, at leading order in $1/N_c$ QCD can be re-expressed in terms of a Lagrangian involving an infinite set of weakly interacting mesons [22]. The precise form of this Lagrangian is not yet known from first principles, but the weak coupling property allows one to relate the coupling constants to observables using tree level calculations and then deduce the values of these observables from experiment.

How well does resonance saturation perform in determining the size of the $O(p^6)$ LECs $C^{\rm r}_i$ is not known at present. The main reason is that very few of these LECs have been determined so far. The purpose of this paper is to derive some constraints on the LECs $C^{\rm r}_i$ obtained by equating the πK scattering

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amplitude in the subthreshold region, as constructed from experimental data in [23], with the chiral expansion calculation up to order p^6 which was performed in [15] (previous work comparing dispersive representations with the chiral expansion up to order p^4 [24] was performed in [25, 26]). Some of the πK subthreshold expansion parameters can be expressed as unsubtracted sum rules. Such expressions allow one to identify resonance contributions from experiment. We will use such results to compare with the same resonance contributions as computed starting from a large N_c type resonance chiral Lagrangian. We will concentrate on a set of contributions arising from vector meson resonances and which can be related to flavor symmetry breaking in the meson multiplet. It is known that the $\pi\pi$ or πK scattering amplitudes receive comparable contributions from vector mesons and from scalar mesons. Describing scalar mesons starting from a resonance chiral Lagrangian presents several difficulties, notably in identifying the properties of the nonet in the chiral limit and in the treatment of the wide resonances. For this reason, we will concentrate here on the vector resonances.

The plan of the paper is as follows. We start by recalling some notation concerning the πK scattering amplitude and some aspects of the correspondence between the expansion parameters around the subthreshold point $t=0,\ s-u=0$ and the $O(p^6)$ LECs. Results concerning the LECs C_1^{r} to C_4^{r} (which are associated with six derivatives chiral operators) are then presented. We next consider πK subthreshold parameters associated with chiral operators involving four derivatives plus one quark mass matrix. In this sector, serious discrepancies are observed between the chiral predictions and the sum rule results. We point out some deficiencies of the resonance model employed in [15] for the relevant $O(p^6)$ LECs and propose a model for the vector meson resonances which implements flavor symmetry breaking (to first order) in a more general way. A phenomenological determination of all the parameters entering this resonance Lagrangian is performed and the complete contribution of order p^6 in terms of the basis of [6] is worked out. Tests of this modelling are performed by comparing with resonance contributions in unsubtracted sum rules. We finally identify a combination of LECs which should be weakly sensitive to the scalar resonance sector and discuss the result.

2 Results on C_1^{r} to C_4^{r}

2.1 Notation

At first, let us recall some standard results and notation concerning the πK scattering amplitude. Assuming isospin symmetry to be exact, πK scattering is described in terms of the two independent isospin amplitudes $F^I(s,t,u)$, with $I = 1/2,\ 3/2$ and the Mandelstam variables, $s,\ t,\ u$, satisfy

$$s+t+u=2\Sigma,\ \Sigma=m_K^2+m_\pi^2\,. \tag{1}$$

Under $s,\ u$ crossing the following relation holds:

$$F^{\frac12}(s,t,u)=-\frac12 F^{\frac32}(s,t,u)+\frac32 F^{\frac32}(u,t,s)\,. \tag{2}$$

It is then convenient to form the two combinations F^+ and F^- which are respectively even and odd under $s,\ u$ crossing,

$$\begin{aligned}F^+(s,t,u)&=\frac13 F^{\frac12}(s,t,u)+\frac23 F^{\frac32}(s,t,u)\\ F^-(s,t,u)&=\frac13 F^{\frac12}(s,t,u)-\frac13 F^{\frac32}(s,t,u)\,.\end{aligned} \tag{3}$$

Under $s,\ t$ crossing F^+ and F^- are simply proportional to the $I=0$ and the $I=1$ $\pi\pi\to K\overline{K}$ amplitudes,

$$\begin{aligned}G^0(t,s,u)&=\sqrt6 F^+(s,t,u)\\ G^1(t,s,u)&=2F^-(s,t,u)\,.\end{aligned} \tag{4}$$

A region where one expects ChPT to apply is around the subthreshold point $t=0,\ s=u=m_K^2+m_\pi^2$. The πK amplitude can be characterized in the neighborhood of this point by performing an expansion in powers of t and $s-u$ [27]. The subthreshold coefficients $C_{ij}^{\pm}$ are dimensionless quantities defined from this expansion

$$\begin{aligned}F^+(s,t,u)&=\sum_{ij} C_{ij}^+\,\frac{t^i\nu^{2j}}{m_{\pi^+}^{2i+2j}}\,,\\ \frac{F^-(s,t,u)}{\nu}&=\sum_{ij} C_{ij}^-\,\frac{t^i\nu^{2j}}{m_{\pi^+}^{2i+2j+1}}\,,\end{aligned} \tag{5}$$

with

$$\nu=\frac{s-u}{4m_K}\,. \tag{6}$$

2.2 Chiral $O(p^6)$ tree level contributions to the subthreshold coefficients

The contributions at tree level from the $O(p^6)$ chiral Lagrangian to the subthreshold coefficients have been worked out in [15] and can be found explicitly in this reference. We will discuss what can be learned about the $O(p^6)$ LECs C_i^{r} from these expressions. Let us begin by noting some general features of the correspondence between the subthreshold coefficients and the LECs. At first, the coefficients such that

$$C_{ij}^+:\ i+2j\ge4,\quad C_{ij}^-:\ i+2j\ge3 \tag{7}$$

get no contribution at all from the $O(p^6)$ LECs. This implies that the chiral expressions at order p^6 for these coefficients obey convergent unsubtracted dispersions relations. As a simple example C_{02}^+ can be written as (which is easily derived from (19) below),

$$C_{02}^+\big|_{p^4+p^6}=\frac{32m_K^4m_\pi^4}{\pi}\int_{m_+^2}^\infty \mathrm{d}s'\,\frac{\operatorname{Im}F^+(s',0)_{p^4+p^6}}{(s'-\Sigma)^5} \tag{8}$$

(with $m_+=m_K+m_\pi$). In this expression one can compute $\operatorname{Im}F^+(s',0)_{p^4+p^6}$ by expanding over partial waves and, for each partial-wave amplitude, using the chiral expansion of the unitarity relation

$$\begin{aligned}&\operatorname{Im} f_l^I(s')_{p^4+p^6}\\ &=\frac{\sqrt\lambda}{s} f_l^I(s')_{p^2}\left[f_l^I(s')_{p^2}+2\mathrm{Re} f_l^I(s')_{p^4}\right].\end{aligned} \tag{9}$$

In this manner, we could reproduce precisely the numerical result $C^+_{02} = 0.23$ obtained in [15].

Next, the chiral expressions for the set of coefficients which satisfy

$$C^+_{ij} : \; i+2j = 3, \quad C^-_{ij} : \; i+2j = 2 \tag{10}$$

involve the four LECs C^r_1, C^r_2, C^r_3, C^r_4 [15] which are associated with the following four chiral Lagrangian terms (the definitions of the various chiral building blocks u_μ, $h_{\lambda\nu}$ etc. which appear below can be found, for instance, in [6])

$$\begin{aligned}
O_1 &= \langle u_\mu\, u^\mu\, h_{\lambda\nu}\, h^{\lambda\nu}\rangle \\
O_2 &= \langle u_\mu\, u^\mu\rangle \,\langle h_{\lambda\nu}\, h^{\lambda\nu}\rangle \\
O_3 &= \langle h_{\mu\nu}\, u_\rho\, h^{\mu\nu}\, u^\rho\rangle \\
O_4 &= \langle h_{\mu\nu}\,(u_\rho\, h^{\mu\rho}\, u^\nu + u_\nu\, h^{\mu\rho}\, u_\rho)\rangle \; .
\end{aligned} \tag{11}$$

These terms contain six derivatives and do not involve quark masses. We will discuss below the determination of these LECs obtained from the subthreshold πK amplitudes as well as from $\pi\pi$ amplitudes.

We next consider the subthreshold coefficients which satisfy

$$C^+_{ij} : \; i+2j = 2, \quad C^-_{ij} : \; i+2j = 1 \; , \tag{12}$$

i.e. the three coefficients C^+_{20}, C^+_{01}, C^-_{10}. Their chiral expansions involve, in addition to C^r_1, C^r_2, C^r_4 the eight LECs $C^r_5 \cdots C^r_8$, $C^r_{10}, \cdots, C^r_{13}$ and the three LECs C^r_{22}, C^r_{23} C^r_{25}. We reproduce the corresponding Lagrangian terms below for the convenience of the reader:

$$\begin{aligned}
&O_5 = \left\langle (u_\mu u^\mu)^2\, \chi_+ \right\rangle \; , \quad O_6 = \left\langle (u_\mu u^\mu)^2 \right\rangle \langle \chi_+\rangle \; , \\
&O_7 = \langle u_\mu u^\mu\rangle \,\langle u_\nu u^\nu \chi_+\rangle \; , \quad O_8 = \langle u_\mu u^\mu u_\nu \chi_+ u^\nu\rangle \; , \\
&O_{10} = \langle \chi_+ u_\mu u_\nu u^\mu u^\nu\rangle \; , \quad O_{11} = \langle\chi_+\rangle \,\langle u_\mu u_\nu u^\mu u^\nu\rangle \; , \\
&O_{12} = \langle h_{\mu\nu}\, h^{\mu\nu}\, \chi_+\rangle \; , \quad O_{13} = \langle h_{\mu\nu}\, h^{\mu\nu}\rangle \,\langle\chi_+\rangle \; , \\
&O_{22} = \mathrm{i}\,\langle \chi_- \,\{h_{\mu\nu}, u^\mu u^\nu\}\rangle \; , \quad O_{23} = \mathrm{i}\,\langle \chi_- h_{\mu\nu}\rangle \,\langle u^\mu u^\nu\rangle \; , \\
&O_{25} = \mathrm{i}\,\langle h_{\mu\nu} u^\mu \chi_- u^\nu\rangle \; .
\end{aligned} \tag{13}$$

These terms contain four derivatives and a single insertion of the quark mass matrix. The information provided by the πK amplitude is not sufficient to determine separately all these LECs. Previously, the LEC C_{12} (as well as the LEC C_{34}) have been determined based on the $\Delta S = 1$ scalar form-factor [28] by combining the chiral $O(p^6)$ calculations of [14] with the dispersive construction method of [29]. Constraints on C_{12} and C_{13} have also been obtained from $\Delta S = 0$ scalar formfactors [14].

Finally, the coefficients with $i+2j = 0, 1$: C^+_{00}, C^+_{10}, C^-_{00} involve thirteen more $O(p^6)$ LECs among those associated with chiral Lagrangian terms containing two or three insertions of the quark mass matrix.

2.3 Determination of $C^r_1, \cdots, C^r_4$

The set of subthreshold coefficients defined in (10) constrain the values of the four LECs $C^r_1 \cdots C^r_4$. The relevant formulas from [15] read

$$\begin{aligned}
C^+_{30}\big|_{C_i} &= \frac{1}{2}\,(-7C^r_1 - 32C^r_2 + 2C^r_3 + 10C^r_4)\,\frac{m^6_\pi}{F^4_\pi} \\
C^+_{11}\big|_{C_i} &= 8\,(3C^r_1 + 6C^r_3 - 2C^r_4)\,\frac{m^4_\pi m^2_K}{F^4_\pi} \\
C^-_{20}\big|_{C_i} &= 6\,(-C^r_1 + 2C^r_3 + 2C^r_4)\,\frac{m^5_\pi m_K}{F^4_\pi} \\
C^-_{01}\big|_{C_i} &= 32\,(-C^r_1 + 2C^r_3 + 2C^r_4)\,\frac{m^3_\pi m^3_K}{F^4_\pi} \; .
\end{aligned} \tag{14}$$

The $O(p^4)$ LECs L^r_i contribute to these coefficients only via one-loop diagrams such that one may use for the L_is the numerical values determined at $O(p^4)$. The last two equations (14) involve the same combination of LECs C^r_i. Therefore, we can determine three combinations, for instance $C^r_1 + 4C^r_3$, C^r_2, $C^r_4 + 3C^r_3$.

Independent information is provided by $\pi\pi$ scattering. The $\pi\pi$ amplitude constrains the two combinations $C^r_4 + 3C^r_3$ and $C^r_1 + 4C^r_3 + 2C^r_2$. The numerical values which we quote in Table 1 make use of the expressions from [30, 31]:

$$\begin{aligned}
r^r_5 &= F^2_\pi\,(-8C^r_1 - 16C^r_2 + 10C^r_3 + 14C^r_4) \\
&\quad + \frac{23F^2_\pi}{15360\,\pi^2 m^2_K} + \text{log terms} \\
r^r_6 &= F^2_\pi\,(6C^r_3 + 2C^r_4) + \frac{F^2_\pi}{15360\,\pi^2 m^2_K} + \text{log terms}
\end{aligned} \tag{15}$$

and the numerical values for r^r_5, r^r_6 obtained in [32] from a Roy equations analysis. The right-hand sides of (15) involves a quadratic polynomial in $\log(m^2_K/\mu^2)$ and $\log(m^2_\eta/\mu^2)$ which we have not determined. We have attempted to minimize its influence by performing the matching at a scale $\mu^2 = m_K m_\eta$ before evolving the scale to M_ρ.

Table 1. Results for combinations of $C^r_1(\mu)$ to $C^r_4(\mu)$ with $\mu = 0.77$ GeV in units of 10^{-4} GeV^{-2} derived from the πK subthreshold parameters. Also shown are results based on the $\pi\pi$ amplitude and from a resonance model

input	$C^r_1 + 4C^r_3$	C^r_2	$C^r_4 + 3C^r_3$	$C^r_1 + 4C^r_3 + 2C^r_2$
$\pi K : C^+_{30}, C^+_{11}, C^-_{20}$	20.7 ± 4.9	-9.2 ± 4.9	9.9 ± 2.5	2.3 ± 10.8
$\pi K : C^+_{30}, C^+_{11}, C^-_{01}$	28.1 ± 4.9	-7.4 ± 4.9	21.0 ± 2.5	13.4 ± 10.8
$\pi\pi$			23.5 ± 2.3	18.8 ± 7.2
Resonance model	7.2	-0.5	10.0	6.2

It is of interest to compare these results from those of the resonance saturation model. In the case of $C_1^{\rm r},\cdots,C_4^{\rm r}$ it suffices to consider resonances in the chiral limit as was the case for the $O(p^4)$ LECs [19]. If one uses simply the same Lagrangian as in [19] (which was also used in the πK analysis of [15]) one obtains

$$C_1^{V+S}=\frac{G_V^2}{8M_V^4}-\frac{c_d^2}{4M_S^4}\,,\ C_3^{V+S}=0\,,$$
$$C_2^{V+S}=\frac{c_d^2}{12M_S^4}-\frac{\tilde{c}_d^2}{4M_{S_1}^4}\,,\ C_4^{V+S}=\frac{G_V^2}{8M_V^4}\,. \tag{16}$$

Contributions from resonance Lagrangian terms like

$$\left\langle\nabla^\lambda V_{\lambda\mu}\,[h_{\mu\nu},u^\nu]\right\rangle,\ \left\langle\nabla^\lambda V_{\mu\nu}\,[h_{\mu\lambda},u^\nu]\right\rangle \tag{17}$$

should, in principle, also be considered, but we will not do so here[1]. In discussing such higher derivative terms, it is important to implement proper asymptotic conditions.

Numerical values are shown in Table 1, using the same values for the couplings as in [19], i.e.

$$G_V=53\,\mathrm{MeV},\ c_d=32\,\mathrm{MeV},\ c_m=42\,\mathrm{MeV}. \tag{18}$$

We note that this value of G_V is somewhat smaller than the one which derives from the $\rho\to 2\pi$ width ($G_V\sim 64.1\,\mathrm{MeV}$; see Sect. 3) but was shown to yield good results for the $O(p^4)$ LECs. In the case of C_2, which is OZI suppressed we show, for illustration, the value derived from the OZI violation model A of [34]. The table also shows that the results obtained using C_{20}^- as input and those using C_{01}^- are compatible for $C_1^{\rm r}+4C_3^{\rm r}$ and for $C_2^{\rm r}$ but not quite so for $C_4^{\rm r}+3C_3^{\rm r}$. The error, however, does not take higher order chiral effects into account. The results which use C_{01}^- are compatible with the $\pi\pi$ results. The simplest resonance saturation model is seen to give correct signs and order of magnitudes for the LECs shown in Table 1, but the agreement is certainly not as good as in the case of the $O(p^4)$ couplings.

3 Symmetry breaking in the vector meson chiral Lagrangian revisited

3.1 Observation of some discrepancies

Let us now turn our attention to the three coefficients C_{20}^+, C_{01}^+, and C_{10}^-. As mentioned above, their chiral expansions get tree level contributions from the Lagrangian terms, $O_5,\cdots,O_{13}$ and $O_{22},\cdots,O_{25}$ which contain four derivatives and one quark mass factor. Their chiral expansions also receive $O(p^4)$ tree level contributions involving the LECs L_1, L_2 L_3. In general, in such a situation, the hope is that one may use a resonance model estimate for the $O(p^6)$ LECs and then derive improved determinations for the LECs L_i. This idea was actually followed in the series of papers [11, 13, 35] which used as experimental input the pseudoscalar meson masses, decay constants and the K_{l4} decay form-factors. Using the determination of the chiral coupling constants obtained in these references from this procedure, the three πK subthreshold coefficients can be predicted. The results obtained in [15] are reproduced in Table 2. Looking at Table 2 it is rather striking that there is a serious discrepancy, for all these three subthreshold coefficients, between the chiral predictions and the dispersive calculations.

Table 2. Comparison of the dispersive results for three sub-threshold parameters (last column) with the chiral calculation of [15] at order p^6. The second and third columns display results obtained when the LECs $L_i^{\rm r}(\mu)$ and $C_i^{\rm r}(\mu)$ are set equal to zero at $\mu=0.77\,\mathrm{GeV}$. The fourth column displays the full chiral result from [15]

	$(p^4)_{L_i=0}$	$(p^6)_{L_i=C_i=0}$	$(p^4+p^6)_{\rm total}$	Dispersive
C_{20}^+	0.0255	−0.0254	0.003	0.024 ± 0.006
C_{01}^+	1.673	1.492	3.8	2.07 ± 0.10
C_{10}^-	−0.0253	0.121	0.09	0.31 ± 0.01

3.2 Should one blame the dispersive representations?

A possible explanation for these discrepancies could be that the dispersive calculations are not correct. Let us argue, considering the particular example of C_{01}^+ which is rather simple, that this is unlikely to be the case. One may start with a fixed-t dispersive representation, at $t=0$, of the amplitude $F^+(s,t)$ with two subtractions,

$$F^+(s,0)=c^+(0)+\frac{1}{\pi}\int_{m_+^2}^\infty \mathrm{d}s'\left[\frac{1}{s'-s}+\frac{1}{s'-u}\right.$$
$$\left.-\frac{2(s'-\Sigma)}{\left(s'-m_+^2\right)\left(s'-m_-^2\right)}\right]\mathrm{Im}\,F^+(s',0)\,. \tag{19}$$

The validity of this kind of dispersion relation as well as that of the Froissart bound which ensures convergence can be established in a rigorous manner [36, 37]. From (19) it is straightforward to derive the following sum rule for the subthreshold parameter C_{01}^+:

$$C_{01}^+=\frac{8m_K^2m_\pi^2}{\pi}\int_{m_+^2}^\infty\frac{\mathrm{Im}\,F^+(s',0)}{(s'-\Sigma)^3}\,\mathrm{d}s'\,. \tag{20}$$

(Note that unlike the case of C_{02}^+, this sum rule is useless for deriving the chiral result.) The integrand needed in this sum rule is displayed in Fig. 1. The following remarks can be made. The contributions from the high energy region $\sqrt{s'}\geq 2\,\mathrm{GeV}$ are negligibly small. Most of the contributions are from the S and the P waves; they are concentrated in the region $\sqrt{s'}\lesssim 1\,\mathrm{GeV}$, and there are no numerical difficulties in computing the integral. The S wave in the lower energy range is the part affected with the largest error. In this region, one can compare with ChPT calculations (which up to order six do not depend on the LECs $C_i^{\rm r}$): the difference is of the order of 20% at most and the ChPT result for $\mathrm{Im}\,F^+(s',0)$ tends to be smaller, and

[1] While this paper was being completed a preprint appeared [33] containing a general discussion of resonance Lagrangian terms contributing at order p^6.

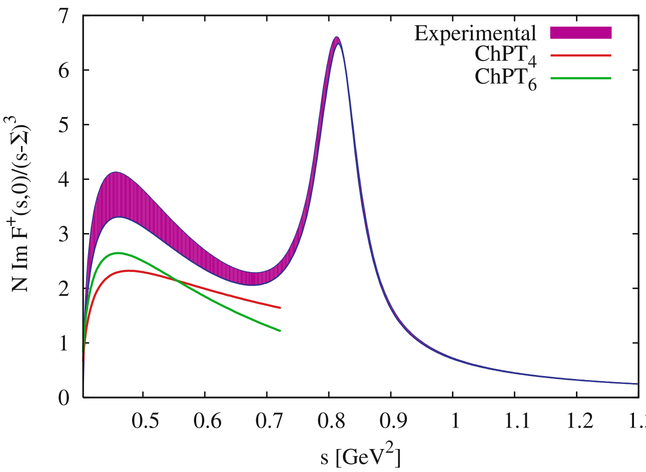

Fig. 1. Integrand to be used in the sum rule, (20)

not larger, than the one derived from experiment. In conclusion, this sum rule seems fairly solid: any reasonable fit to the experimental data of [38, 39] will give a number $C_{01}^{+} \simeq 2$.

3.3 Vector resonance chiral Lagrangian

Another possible cause for the discrepancies revealed in Table 2 could be that the resonance saturation model used to evaluate the LECs, C_i, needs to be improved. The coefficients in Table 2 are associated with $O(p^6)$ operators containing one quark mass factor such that the corresponding LECs are sensitive to flavor symmetry breaking of the light resonances. We will re-examine the case of the vector mesons here, and we follow the approach to first construct a Lagrangian containing the resonance fields and then integrate them out. A convenient method for this construction (see e.g. [40]) is to make use of non-linear representations of the chiral group [41]. For the purpose of generating chiral Lagrangian terms it is also convenient to adopt a homogeneous chiral transformation rule for all the resonances:

$$R \to h[\phi]\, R\, h[\phi]^{\dagger}\,. \tag{21}$$

Such a transformation rule ensures that one can ascribe a definite chiral order to each resonance field. A detailed discussion in connection with the $O(p^4)$ LECs can be found in [19, 42]. In [15] the vector field formalism [42] was used and flavor symmetry breaking is described via the single term

$$\mathcal{L}_V^m = f_\chi \left\langle V_\mu \left[u^\mu, \chi_-\right]\right\rangle\,. \tag{22}$$

This term is the unique one relevant to the $O(p^6)$ LECs because in the vector field formalism the field V_μ has chiral order three. The coupling constant f_χ was determined such as to reproduce the experimental value for the ratio $\Gamma(K^* \to K\pi)/\Gamma(\rho \to \pi\pi)$, which gave

$$f_\chi = -0.025\,. \tag{23}$$

In seeking for an improvement we note that, in this formalism, the symmetry-breaking effects induced from the masses of the vector mesons are absent at order p^6, which seem somewhat unnatural. This suggests us to investigate different formalisms. A discussion of symmetry breaking based on the massive Yang–Mills approach was performed in [43]. We will make use here of the formalism which uses anti-symmetric tensors [2, 19] instead of vector fields[2]. The part which is relevant for the $O(p^4)$ LECs was considered in [19]:

$$\mathcal{L}_{AT}^0 = -\frac{1}{2}\left\langle \nabla^\lambda V_{\lambda\mu} \nabla_\nu V^{\nu\mu}\right\rangle + \frac{1}{4} M_V^2 \left\langle V_{\mu\nu} V^{\mu\nu}\right\rangle + \frac{F_V}{2\sqrt{2}}\left\langle V_{\mu\nu} f_+^{\mu\nu}\right\rangle + \frac{\mathrm{i}\, G_V}{\sqrt{2}}\left\langle V_{\mu\nu} u^\mu u^\nu\right\rangle\,. \tag{24}$$

From (24) one can deduce the chiral order of the resonance field:

$$V_{\mu\nu} \sim O(p^2)\,. \tag{25}$$

As a consequence, the kinetic energy term in (24) is $O(p^6)$, while the other terms are $O(p^4)$. Let us now consider all possible terms which are chiral symmetry-breaking corrections to the terms in (24). Neglecting OZI rule violation, we find that there are six independent such terms which have chiral order six (some of these have been considered also recently in [44]):

$$\begin{aligned}\mathcal{L}_{AT}^m = &\frac{1}{2} e_V^m \left\langle \chi_+ V^{\mu\nu} V_{\mu\nu}\right\rangle + \mathrm{i}\frac{g_{V1}^m}{M_V}\left\langle V^{\mu\nu}\{\chi_+, u_\mu u_\nu\}\right\rangle \\ &+ \mathrm{i}\frac{g_{V2}^m}{M_V}\left\langle V^{\mu\nu} u_\mu \chi_+ u_\nu\right\rangle - \frac{f_\chi}{M_V}\left\langle \nabla_\mu V^{\mu\nu}[\chi_-, u_\nu]\right\rangle \\ &+ \frac{f_{V1}^m}{M_V}\left\langle V_{\mu\nu}\left\{f_+^{\mu\nu}, \chi_+\right\}\right\rangle + \frac{f_{V2}^m}{M_V}\left\langle V_{\mu\nu}\left[f_-^{\mu\nu}, \chi_-\right]\right\rangle\,.\end{aligned} \tag{26}$$

Only the first four of these terms play a role in πK scattering. Instead of a single coupling constant, f_χ, in the vector formalism, one has four independent couplings here: e_V^m, g_{V1}^m, g_{V2}^m, f_χ. Let us now discuss the determination of these couplings from experiment.

3.4 Determination of the vector Lagrangian coupling constants

3.4.1 Determination of e_V^m

First, it is not difficult to determine e_V^m based on the mass relations

$$\begin{aligned} M_\rho^2 &= M_V^2 + 8 e_V^m B_0 \hat{m}\,, \\ M_{K^*}^2 &= M_V^2 + 4 e_V^m B_0\left(m_s + \hat{m}\right)\,. \end{aligned} \tag{27}$$

Isospin breaking is neglected here and we have denoted $\hat{m} = (m_u + m_d)/2$. For the numerics, we can use the results from

[2] In [42] it was shown how these two formalisms can be made to give exactly equivalent results for the $O(p^4)$ LECs. This necessitates that a number of asymptotic constraints for Green's functions, form-factors or scattering amplitudes be implemented.

the chiral expansion at leading order,

$$\begin{aligned} &2B_0\hat{m} = M^2_{\pi^0} = (134.98\,\mathrm{MeV})^2\,, \\ &\frac{m_s}{\hat{m}} = \frac{M^2_{K^+}+M^2_{K^0}-M^2_{\pi^+}}{M^2_{\pi^0}} \simeq 25.90\,, \\ &B_0(m_u-m_d) = M^2_{K^+}-M^2_{K^0}-M^2_{\pi^+}+M^2_{\pi^0} \\ &\qquad\qquad \simeq -0.285\,M^2_{\pi^0}\,. \end{aligned} \tag{28}$$

Using the experimental values of the $K^*(892)$ and the $\rho(770)$ masses we obtain

$$e^m_V \simeq 0.22\,. \tag{29}$$

If, instead, one uses the masses of the $\phi(1020)$ and the $\rho(770)$ mesons one would obtain $e^m_V \simeq 0.24$, suggesting that the error should be reasonably small for this quantity.

3.4.2 Determination of f_χ and f^m_{V2}

As a next step we consider the coupling constant f_χ. In [15] f_χ was related to symmetry breaking in the decays of vectors into two pseudoscalars. Here, we will argue that these decays determine the two couplings g^m_{V1}, g^m_{V2}. Concerning f_χ, a physically plausible estimate can be obtained by relating it to the decay of the $\pi(1300)$ resonance. Let us denote the $\pi(1300)$ nonet matrix by P and consider the Lagrangian,

$$\begin{aligned} \mathcal{L}^{\pi(1300)} = &\frac{1}{2}\langle\nabla^\mu P\nabla_\mu P\rangle - \frac{1}{2}M_P^2\langle P^2\rangle + \mathrm{i}d_m\langle P\chi_-\rangle \\ &+\mathrm{i}G'_V\langle\nabla_\mu V^{\mu\nu}[P,u_\nu]\rangle + \mathrm{i}G''_V\left\langle V_{\mu\nu}\left[f_-^{\mu\nu},P\right]\right\rangle\,. \end{aligned} \tag{30}$$

This extends the Lagrangian considered in [19] by the last two terms proportional to G'_V and G''_V respectively and which have chiral order equal to six. Integrating out the $\pi(1300)$ meson, one finds that the couplings f_χ and f^m_{V2} which were appearing in (26) are proportional respectively to G'_V and G''_V:

$$f_\chi = G'_V\frac{d_m M_V}{M_P^2}, \quad f^m_{V2} = G''_V\frac{d_m M_V}{M_P^2}\,. \tag{31}$$

The coupling d_m was introduced in [19]. It can be estimated by appealing to a chiral super-convergence sum rule associated with the correlator of two scalar currents minus the correlator of two pseudoscalar currents [2]. Saturating the sum rule from the contributions of the pion, the $\pi(1300)$ and the $a_0(980)$, one gets the relation

$$8d_m^2 + F_0^2 - 8c_m^2 = 0\,. \tag{32}$$

Using the value $c_m \simeq 42\,\mathrm{MeV}$ which was obtained [19] then gives

$$d_m \simeq 26\,\mathrm{MeV}\,. \tag{33}$$

The coupling G'_V can be related to the decay amplitude $\pi(1300)\to\rho\pi$,

$$\Gamma_{\pi(1300)\to\rho\pi} = \frac{2(G'_V)^2 p^3_{cm}}{\pi F_\pi^2}\,. \tag{34}$$

The total width of the $\pi(1300)$ is known to be rather large but has not actually been very precisely determined (the PDG [45] quotes a range of values between 200 and 600 MeV). For definiteness, let us use the result obtained in [46] where one also finds the $\rho\pi$ decay mode to be the dominant one:

$$\Gamma_{\pi(1300)\to\rho\pi} \simeq 268\pm 50\,\mathrm{MeV}. \tag{35}$$

This gives the estimate

$$|G'_V| \simeq 0.23\,, \tag{36}$$

yielding

$$|f_\chi| \simeq 2.8\,10^{-3}\,. \tag{37}$$

This value is one order of magnitude smaller than the one obtained in [15]. One consequence concerns the lifetime of the πK atom which receives a contribution (via resonance saturation of the LECs C_i) which is quadratic in f_χ. If one uses the numerical value (37) for f_χ, the size of the $O(p^6)$ contribution to the lifetime is rather small (see the detailed discussion in [47]).

A somewhat different approach is to consider the 3-point correlation function $\langle VAP\rangle$, model it in terms of a finite number of resonances, and constrain the coupling constants in order to enforce the proper QCD asymptotic conditions [48, 49]. This was reconsidered recently by Cirigliano et al. [50] who improved on earlier work by including the $\pi(1300)$ nonet in the construction together with the vector, axial-vector and pion multiplets. In this manner, they have obtained a determination of the $\pi(1300)$ couplings G'_V, G''_V in terms of the vector and axial-vector resonance masses,

$$G'_V = -\frac{\sqrt{M_A^2-M_V^2}}{2M_A}, \quad G''_V = -\frac{\sqrt{M_A^2-M_V^2}}{8M_A}\,. \tag{38}$$

Using $M_A = \sqrt{2}M_V$ this gives

$$f_\chi \simeq -4.2\,10^{-3}, \quad f^m_{V2} \simeq -1.1\,10^{-3}\,. \tag{39}$$

This method provides a determination of f_χ which is in reasonably good agreement with the one based on the $\pi(1300)$ decay width and gives also the sign as well as a determination of the coupling f^m_{V2}.

3.4.3 Determination of g^m_{V1}

The coupling g^m_{V1} can be determined from the decay amplitudes of vector mesons into two pseudoscalars. The decay amplitudes have the following form:

$$\mathcal{T}(V_a\to\phi_b\,\phi_c) = M_{V_a}\epsilon\cdot(p_1-p_2)\,T_{abc}\,. \tag{40}$$

Correspondingly, the decay width is given by

$$\Gamma(V_a\to\phi_b\,\phi_c) = |T_{abc}|^2\,\frac{p^3_{cm}}{6\pi}\,. \tag{41}$$

Using the Lagrangian (26) these amplitudes get expressed as a function of two combinations of the couplings g^m_{V1}, g^m_{V2} and f_χ for which we introduce the notation

$$\begin{aligned} \hat{g}^m_{V1} &= g^m_{V1} + \frac{1}{2}f_\chi \\ \hat{g}^m_{V2} &= g^m_{V2} + f_\chi\,. \end{aligned} \tag{42}$$

The amplitude for $\rho^+ \to \pi^+\pi^0$, at first, reads

$$T_{\rho^+\to\pi^+\pi^0} = \frac{1}{F_\pi^2}\, G_V^{\rm eff}\,,$$
$$G_V^{\rm eff} = G_V + \frac{4\sqrt{2}\hat{m}B_0}{M_V}\left(2\,\hat{g}_{V1}^m + \hat{g}_{V2}^m\right)\,. \tag{43}$$

Using the experimental values for the mass $m_\rho = 775.5 \pm 0.5\,\mathrm{MeV}$ and the width $\Gamma = 150.2 \pm 2.4\,\mathrm{MeV}$ from [45] gives

$$G_V^{\rm eff} \simeq 65.8\,\mathrm{MeV}\,. \tag{44}$$

Next, we consider the decays $K^* \to K\pi$ and $\phi \to K\bar{K}$.

$$\begin{aligned}
T_{K^{*+}\to K^0\pi^+} &= \frac{\sqrt{2}}{2F_{K^0}F_{\pi^+}}\left\{G_V^{\rm eff} + \frac{4\sqrt{2}\,\hat{g}_{V1}^m}{M_V}B_0\,(m_s-\hat{m})\right.\\
&\quad\left. + \frac{2\sqrt{2}\,(\,\hat{g}_{V1}^m - \hat{g}_{V2}^m)}{M_V}B_0(m_u-m_d)\right\}\,,\\
T_{K^{*+}\to K^+\pi^0} &= \frac{1}{2F_{K^+}F_{\pi^0}}\left\{G_V^{\rm eff} + \frac{4\sqrt{2}\,\hat{g}_{V1}^m}{M_V}B_0\,(m_s-\hat{m})\right.\\
&\quad\left. + \frac{2\sqrt{2}\,(\,\hat{g}_{V1}^m + \hat{g}_{V2}^m)}{M_V}B_0(m_u-m_d)\right\}\,,\\
T_{\phi\to K^+K^-} &= -\frac{\sqrt{2}\,e^2F_V}{6M_\phi^2}\,\frac{2M_\rho^2-M_\phi^2}{M_\rho^2-M_\phi^2}\\
&\quad + \frac{\sqrt{2}}{2F_{K^+}^2}\left\{G_V^{\rm eff} + \frac{8\sqrt{2}\,\hat{g}_{V1}^m}{M_V}B_0\,(m_s-\hat{m})\right.\\
&\quad\left. + \frac{2\sqrt{2}\,\hat{g}_{V2}^m}{M_V}B_0(m_u-m_d)\right\}\,,\\
T_{\phi\to K^0\overline{K}^0} &= \frac{\sqrt{2}\,e^2F_V}{6M_\phi^2}\,\frac{M_\phi^2}{M_\rho^2-M_\phi^2}\\
&\quad + \frac{\sqrt{2}}{2F_{K^0}^2}\left\{G_V^{\rm eff} + \frac{8\sqrt{2}\,\hat{g}_{V1}^m}{M_V}B_0\,(m_s-\hat{m})\right.\\
&\quad\left. - \frac{2\sqrt{2}\,\hat{g}_{V2}^m}{M_V}B_0(m_u-m_d)\right\}\,.
\end{aligned} \tag{45}$$

These expressions include isospin-breaking contributions proportional to $m_u - m_d$ and those proportional to e^2F_V induced by the coupling of the neutral vector mesons to the photon. We have also taken into account the influence of wave-function renormalization of the pseudoscalar mesons. If we ignore isospin breaking, i.e. set $m_u = m_d$, then the decay amplitudes (45) no longer depend on $\hat{g}_{V2}^m$, which allows us to determine $\hat{g}_{V1}^m$. Combining the experimental values [45] for the K^{*+} and K^{*0} decay widths into $K\pi$ we obtain

$$\hat{g}_{V1}^m \simeq 6.0 \times 10^{-3}. \tag{46}$$

If one uses the ϕ decay widths into K^+K^- and $K^0\bar{K}^0$ instead, one obtains a smaller but not very different value,

$$\hat{g}_{V1}^m \simeq 4.3 \times 10^{-3}. \tag{47}$$

From these two results one can infer $\hat{g}_{V1}^m = (5.2 \pm 1.5) \times 10^{-3}$.

3.4.4 Determination of g_{V2}^m

Finally, we have to determine g_{V2}^m. The results of the previous subsection shows that if one forms isospin-breaking combinations,

$$\begin{aligned}
&T_{K^{*+}\to K^0\pi^+} - \sqrt{2}T_{K^{*+}\to K^+\pi^0}\,,\\
&T_{\phi\to K^+K^-} - T_{\phi\to K^0\bar{K}^0}\,,
\end{aligned} \tag{48}$$

the coupling g_{V2}^m is the only one which contributes. In practice, however, it turns out not to be possible to determine g_{V2}^m in this way. Precise experimental information exists for isospin violation in ϕ decays but, in this case, there are significant electromagnetic contributions as well, which are difficult to evaluate. Further amplitudes which vanish in the isospin limit are $\omega \to \pi^+\pi^-$ and $\rho^+ \to \pi^+\eta$. These amplitudes have the following expressions:

$$\begin{aligned}
T_{\omega\to\pi^+\pi^-} &= \frac{G_V}{F_\pi^2\left(M_\omega^2 - M_\rho^2\right)}\\
&\quad \times \left\{\frac{m_u-m_d}{m_s-\hat{m}}\left(M_{K^*}^2 - M_\rho^2\right) + \frac{e^2F_V^2}{3}\right\}\\
&\quad + \frac{2\sqrt{2}}{F_\pi^2}\,\frac{(2\,\hat{g}_{V1}^m - \hat{g}_{V2}^m)}{M_V}B_0(m_u-m_d)\,,\\
T_{\rho^+\to\pi^+\eta} &= \frac{\sqrt{3}\,G_V^{\rm eff}}{4F_\pi F_\eta}\,\frac{(m_u-m_d)}{(m_s-\hat{m})}\\
&\quad + \frac{2\sqrt{2}}{\sqrt{3}F_\pi F_\eta}\,\frac{\hat{g}_{V2}^m}{M_V}B_0(m_u-m_d)\,.
\end{aligned} \tag{49}$$

In these cases the contribution proportional to g_{V2}^m can be estimated to be relatively small, so that it is again difficult to precisely extract its value.

The coupling $\hat{g}_{V2}^m$ appears in the amplitude $\rho \to K\overline{K}$, as one can see from the expression

$$\begin{aligned}
&T\left(\rho^+ \to K^+\overline{K}^0\right) \\
&\quad = \frac{1}{\sqrt{2}F_K^2}\left\{G_V^{\rm eff} + \frac{4\sqrt{2}\,\hat{g}_{V2}^m}{M_V}B_0\,(m_s-\hat{m})\right\}\,.
\end{aligned} \tag{50}$$

From an experimental point of view, one can hope to determine this amplitude from the τ decay process $\tau \to K\overline{K}\nu_\tau$. It is customary to approximate the dynamics of τ hadronic decays as proceeding via a few resonances [51]. In the case of the $K\overline{K}$ channel, the $\rho(770)$ and the $\rho(1450)$ resonances can contribute [52]. The resonance $\rho(1450)$ has a rather small coupling to $K\overline{K}$ [45] and its contribution is also suppressed by phase-space such that it seems a plausible approximation to saturate the integrated $\tau \to K\overline{K}\nu_\tau$ decay width from just the ρ contribution. In order to compute this decay width from our resonance model we first introduce the charged vector current matrix element which, in the isospin limit, involves a single form-factor:

$$\begin{aligned}
&\left\langle K^-(p_1)K^0(p_2)|\bar{d}\gamma^\mu u|0\right\rangle = (p_1-p_2)^\mu F_V^K(s),\\
&s = (p_1+p_2)^2\,.
\end{aligned} \tag{51}$$

Computing the form-factor from our effective Lagrangian, we obtain

$$F_V^K(s) = 1+\frac{F_V}{F_K^2}\left(G_V^{\mathrm{eff}}+\frac{4\sqrt{2}\,\hat{g}_{V2}^m}{M_V}B_0\left(m_s-\hat{m}\right)\right)\frac{s}{M_V^2-s}\,. \tag{52}$$

The τ decay rate into $K\overline{K}\nu_\tau$ has the following expression:

$$\Gamma_{K\overline{K}} = V_{ud}^2\frac{G_{\mathrm{F}}^2M_\tau^5}{768\pi^3}\int_{4m_K^2}^{M_\tau^2}\frac{\mathrm{d}s}{M_\tau^2}\left(1-\frac{4m_K^2}{s}\right)^{\frac{3}{2}} \times\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right)\left|F_V^K(s)\right|^2\,. \tag{53}$$

In practice, the formula (52), which is obtained from a tree level calculation, does not account for the ρ meson width. One may account for this effect in a phenomenological way by replacing M_V^2 in the propagator in (52) by $M_V^2-\mathrm{i}M_V\Gamma(s)$. In the energy range relevant for τ decay we retain the contributions to the ρ width arising from the $\pi\pi$ and the $K\overline{K}$ channels as well as the 4π channel, simply approximated as $\omega\pi$ which gives, in the region $s\geq 4m_K^2$,

$$M_V\Gamma(s) = \frac{M_V^2\Gamma_V}{\sqrt{s}}\left[\left(\frac{s-4m_\pi^2}{M_V^2-4m_\pi^2}\right)^{\frac{3}{2}}+\frac{1}{2}\left(\frac{s-4m_K^2}{M_V^2-4m_\pi^2}\right)^{\frac{3}{2}}\right] +\frac{G_{\omega\rho\pi}^2}{4\pi}\frac{\left[\left(s-(M_\omega+m_\pi)^2\right)\left(s-(M_\omega-m_\pi)^2\right)\right]^{\frac{3}{2}}}{24s}\,. \tag{54}$$

The coupling constant $G_{\omega\rho\pi}$ may be estimated using vector meson dominance and the experimental value of the $\omega\to\gamma\pi$ width [53]

$$\frac{G_{\omega\rho\pi}^2}{4\pi}\simeq 24\,\mathrm{GeV}^{-2}\,. \tag{55}$$

Using the expression (54) for the imaginary part of the ρ meson propagator and the experimental value [45] of the $\tau\to K\overline{K}\nu$ decay rate $R=(15.4\pm1.6)\times10^{-3}$ we obtain

$$\hat{g}_{V2}^m\simeq 0.015\,. \tag{56}$$

Ignoring completely the ρ width gives a larger value, $\hat{g}_{V2}^m\simeq 0.022$. Alternatively, one may estimate $\hat{g}_{V2}^m$ by making use of an asymptotic constraint, namely imposing that the form-factor $F_V^K(s)$ goes as $1/s$ asymptotically. This yields a somewhat smaller value $\hat{g}_{V2}^m\simeq 0.011$. This discussion allows us to estimate that the error on the estimate (56) should be of the order of 50%, i.e. $\hat{g}_{V2}^m=0.015\pm0.007$.

3.4.5 Determination of f_{V1}^m

Finally, let us consider f_{V1}^m. This parameter controls flavor symmetry breaking in the matrix elements of the vector current between a vector meson and the vacuum,

$$F_{K^*}-F_\rho=\frac{8\sqrt{2}\,f_{V1}^m}{M_V}B_0\left(m_s-\hat{m}\right)\,. \tag{57}$$

We can extract the relevant information from the τ decay processes $\tau\to\rho^-\nu_\tau$ and $\tau\to K^{*-}\nu_\tau$. Using the experimental results from [45], we obtain

$$F_\rho=146.3\pm1.2\,\mathrm{MeV},\quad F_{K^*}=155.1\pm4.0\,\mathrm{MeV}\,, \tag{58}$$

from which we finally deduce

$$f_{V1}^m=0.0027\pm0.0013\,. \tag{59}$$

3.5 Vector meson contributions to the LECs

Let us now integrate out the vector meson from the Lagrangian of (24) and (26) and consider the $O(p^6)$ chiral Lagrangian terms which are generated. One finds

$$\begin{aligned}
\mathcal{L}_{AT}^{(6)} = {} & \frac{G_V^2}{4M_V^4}\left\langle\nabla_\lambda[u^\lambda,u^\mu]\nabla_\nu\left[u^\nu,u_\mu\right]\right\rangle \\
& -\left(\frac{e_V^mG_V^2}{2M_V^4}-\frac{\sqrt{2}G_V\,g_{V1}^m}{M_V^3}\right)\left\langle[u_\mu,u_\nu]u^\mu u^\nu\chi_+\right\rangle \\
& +\frac{G_V\,g_{V2}^m}{\sqrt{2}M_V^3}\left\langle[u_\mu,\,u_\nu]u^\mu\chi_+u^\nu\right\rangle \\
& -\frac{G_Vf_\chi}{\sqrt{2}M_V^3}\mathrm{i}\left\langle\nabla_\mu[\chi_-,\,u_\nu][u^\mu,\,u^\nu]\right\rangle \\
& -\frac{G_VF_V}{2M_V^4}\mathrm{i}\left\langle\nabla_\nu[u^\nu,u^\mu]\nabla^\lambda f_{+\lambda\mu}\right\rangle \\
& -\frac{F_V^2}{4M_V^4}\left\langle\nabla^\lambda f_{+\lambda\mu}\nabla_\nu f_+^{\nu\mu}\right\rangle \\
& -\frac{F_Vf_\chi}{\sqrt{2}M_V^3}\left\langle f_{+\mu\nu}\nabla^\mu\left[\chi_-,u^\nu\right]\right\rangle \\
& +\left(\frac{F_VG_Ve_V^m}{2M_V^4}-\frac{2G_V\,f_{V1}^m}{\sqrt{2}M_V^3}-\frac{F_V\,g_{V1}^m}{\sqrt{2}M_V^3}\right) \\
& \times\mathrm{i}\left\langle f_{+\mu\nu}\left\{\chi_+,u^\mu u^\nu\right\}\right\rangle-\frac{F_V\,g_{V2}^m}{\sqrt{2}M_V^3}\mathrm{i}\left\langle f_{+\mu\nu}u^\mu\chi_+u^\nu\right\rangle \\
& +\left(\frac{e_V^mF_V^2}{4M_V^4}-\frac{2F_V\,f_{V1}^m}{\sqrt{2}M_V^3}\right)\left\langle\chi_+f_{+\mu\nu}f_+^{\mu\nu}\right\rangle \\
& -\frac{F_V\,f_{V2}^m}{\sqrt{2}M_V^3}\left\langle f_{+\mu\nu}\left[f_-^{\mu\nu},\chi_-\right]\right\rangle \\
& -\frac{2G_V\,f_{V2}^m}{\sqrt{2}M_V^3}\mathrm{i}\left\langle f_{-\mu\nu}[\chi_-,u^\mu u^\nu]\right\rangle\,.
\end{aligned} \tag{60}$$

In the vector field formalism one term, proportional to f_χ^2, is generated which does not appear in (60). In the spirit of [42] we may simply add this term here[3]:

$$\mathcal{L}_V^{(6)}=-\frac{f_\chi^2}{2M_V^2}\left\langle[u_\mu,\chi_-][u^\mu,\chi_-]\right\rangle\,. \tag{61}$$

In this way, we recover exactly the results of [15] if we set the extra coupling constants in our vector Lagrangian

[3] Alternatively, one may describe spin one resonances in terms of a *pair* of fields $V_{\mu\nu}$ and V_μ. A more detailed discussion of this framework will be presented elsewhere [54].

equal to zero. Next, we can expand the chiral Lagrangian terms over the canonical $O(p^6)$ basis established in [6]. After some calculation, we obtain contributions to 45 different LECs

$$
\begin{aligned}
C_1^V &= \frac{G_V^2}{8M_V^4}, \\
C_4^V &= \frac{G_V^2}{8M_V^4}, \\
C_5^V &= -\frac{G_V g_{V2}^m}{\sqrt{2}M_V^3}, \\
C_8^V &= \frac{e_V^m G_V^2}{2M_V^4} - \frac{\sqrt{2}G_V g_{V1}^m}{M_V^3}, \\
C_{10}^V &= -\frac{e_V^m G_V^2}{2M_V^4} + \frac{\sqrt{2}G_V g_{V1}^m}{M_V^3} + \frac{G_V g_{V2}^m}{\sqrt{2}M_V^3}, \\
C_{22}^V &= \frac{G_V^2}{16M_V^4} + \frac{G_V f_\chi}{2\sqrt{2}M_V^3}, \\
C_{24}^V &= \frac{1}{n}\frac{G_V^2}{4M_V^4}, \\
C_{25}^V &= -\frac{3G_V^2}{8M_V^4} - \frac{G_V f_\chi}{\sqrt{2}M_V^3}, \\
C_{26}^V &= \frac{G_V^2}{4M_V^4} - \frac{1}{n^2}\frac{G_V^2}{2M_V^4} + \frac{G_V f_\chi}{\sqrt{2}M_V^3} + \frac{f_\chi^2}{M_V^2}, \\
C_{27}^V &= -\frac{1}{n}\frac{G_V^2}{4M_V^4} + \frac{1}{n^2}\frac{G_V^2}{2M_V^4}, \\
C_{28}^V &= \frac{1}{n^2}\frac{G_V^2}{8M_V^4}, \\
C_{29}^V &= -\frac{G_V^2}{8M_V^4} - \frac{1}{n^2}\frac{G_V^2}{4M_V^4} - \frac{G_V f_\chi}{\sqrt{2}M_V^3} - \frac{f_\chi^2}{M_V^2}, \\
C_{30}^V &= \frac{1}{n^2}\frac{G_V^2}{4M_V^4}, \\
C_{40}^V &= -\frac{G_V^2}{8M_V^4}, \\
C_{42}^V &= -\frac{G_V^2}{8M_V^4}, \\
C_{44}^V &= \frac{G_V^2}{4M_V^4}, \\
C_{48}^V &= -\frac{G_V^2}{8M_V^4}, \\
C_{50}^V &= \frac{G_V F_V}{4M_V^4} + \frac{f_\chi F_V}{\sqrt{2}M_V^3}, \\
C_{51}^V &= -\frac{G_V^2}{4M_V^4} + \frac{G_V F_V}{4M_V^4} + \frac{f_\chi F_V}{\sqrt{2}M_V^3}, \\
C_{52}^V &= -\frac{G_V F_V}{4M_V^4} - \frac{f_\chi F_V}{\sqrt{2}M_V^3}, \\
C_{53}^V &= -\frac{G_V F_V}{8M_V^4} - \frac{3F_V^2}{16M_V^4} - \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{55}^V &= \frac{G_V F_V}{8M_V^4} + \frac{3F_V^2}{16M_V^4} + \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{56}^V &= -\frac{G_V F_V}{4M_V^4} + \frac{3F_V^2}{8M_V^4} - \frac{f_\chi F_V}{\sqrt{2}M_V^3}, \\
C_{57}^V &= \frac{G_V F_V}{2M_V^4} + \frac{F_V^2}{8M_V^4} + \frac{\sqrt{2}f_\chi F_V}{M_V^3}, \\
C_{59}^V &= -\frac{G_V F_V}{8M_V^4} - \frac{F_V^2}{4M_V^4} - \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{61}^V &= \frac{e_V^m F_V^2}{4M_V^4} - \frac{\sqrt{2}F_V\, f_{V1}^m}{M_V^3}, \\
C_{63}^V &= -\frac{\sqrt{2}\, f_{V1}^m G_V}{M_V^3} + \frac{e_V^m F_V G_V}{2M_V^4} - \frac{F_V\, g_{V1}^m}{\sqrt{2}M_V^3}, \\
C_{65}^V &= -\frac{F_V\, g_{V2}^m}{\sqrt{2}M_V^3}, \\
C_{66}^V &= \frac{G_V^2}{8M_V^4}, \\
C_{69}^V &= -\frac{G_V^2}{8M_V^4}, \\
C_{70}^V &= -\frac{G_V^2}{8M_V^4} - \frac{G_V F_V}{8M_V^4} + \frac{F_V^2}{8M_V^4} - \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{72}^V &= \frac{G_V F_V}{8M_V^4} - \frac{F_V^2}{8M_V^4} + \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{73}^V &= \frac{G_V F_V}{4M_V^4} - \frac{F_V^2}{8M_V^4} + \frac{f_\chi F_V}{\sqrt{2}M_V^3}, \\
C_{74}^V &= -\frac{G_V^2}{4M_V^4}, \\
C_{76}^V &= -\frac{G_V F_V}{8M_V^4} + \frac{F_V^2}{16M_V^4} - \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{78}^V &= \frac{G_V F_V}{8M_V^4} + \frac{F_V^2}{4M_V^4} + \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{79}^V &= -\frac{G_V F_V}{8M_V^4} + \frac{F_V^2}{8M_V^4} - \frac{f_\chi F_V}{2\sqrt{2}M_V^3}, \\
C_{82}^V &= -\frac{G_V F_V}{16M_V^4} - \frac{F_V^2}{16M_V^4} - \frac{f_\chi F_V}{4\sqrt{2}M_V^3} - \frac{f_{V2}^m F_V}{\sqrt{2}M_V^3}, \\
C_{83}^V &= \frac{3G_V^2}{16M_V^4} + \frac{f_\chi G_V}{2\sqrt{2}M_V^3} - \frac{\sqrt{2}\, f_{V2}^m G_V}{M_V^3}, \\
C_{87}^V &= \frac{F_V^2}{8M_V^4}, \\
C_{88}^V &= -\frac{G_V F_V}{4M_V^4} - \frac{f_\chi F_V}{\sqrt{2}M_V^3}, \\
C_{89}^V &= \frac{F_V^2}{2M_V^4} + \frac{G_V F_V}{4M_V^4}, \\
C_{90}^V &= -\frac{f_\chi F_V}{\sqrt{2}M_V^3}, \\
C_{92}^V &= \frac{F_V^2}{M_V^4}, \\
C_{93}^V &= -\frac{F_V^2}{4M_V^4}\,.
\end{aligned}
\tag{62}
$$

In these formulas n stands for the number of flavors and should be set to $n = 3$.

These results can be verified to agree with the ones obtained in [33] when retaining the same resonance coupling constants as in our Lagrangian. The correspondence in the notation between the coupling constants appearing in our (26) and those in [33] is as follows:

$$\begin{aligned} e_V^m \frac{F_V^2}{2M_V^4} &= \overline{\lambda}_6^{\mathrm{VV}} + \overline{\lambda}^{\mathrm{SVV}} & g_{V1}^m \frac{F_V}{M_V^3} &= \overline{\lambda}_1^{\mathrm{SV}}\,, \\ g_{V2}^m \frac{F_V}{M_V^3} &= -\overline{\lambda}_2^{\mathrm{SV}}\,, & f_\chi \frac{F_V}{M_V^3} &= \overline{\lambda}_1^{\mathrm{PV}}\,, \\ f_{V2}^m \frac{F_V}{M_V^3} &= -\overline{\lambda}_2^{\mathrm{PV}} - \frac{1}{2}\overline{\lambda}_1^{\mathrm{PV}}\,, & f_{V1}^m \frac{F_V}{M_V^3} &= \overline{\lambda}_3^{\mathrm{SV}}\,. \end{aligned} \tag{63}$$

These relationships may be derived by making a field redefinition on the scalar and pseudoscalar resonance fields used in [33]:

$$S \to \tilde{S} + c_m \frac{\chi_+}{M_S^2}, \qquad P \to \tilde{P} + \mathrm{i} d_m \frac{\chi_-}{M_P^2}\,. \tag{64}$$

We note that the terms proportional to f_χ^2 in C_{25}^V and C_{26}^V which are generated in the V-formalism but not directly in the AT-formalism have not been considered in [33].

3.5.1 Resonance saturation versus experiment for C_{61}

Only one of the LECs which appear in (62) (except for C_1 and C_4) has actually been determined from experiment. Let us consider the 2-point correlator of two vector currents,

$$\begin{aligned} &\mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}px} \left\langle 0 \left| T\left(V_\mu^{ij}(x) V_\nu^{ji}(0)\right) \right| 0 \right\rangle \\ &\quad = \left(p_\mu p_\nu - p^2 g_{\mu\nu}\right) \Pi^{ij}(p^2) + g_{\mu\nu} p^2\, \Pi_0^{ij}(p^2)\,, \end{aligned} \tag{65}$$

with

$$V_\mu^{ij}(x) = \bar{\psi}^i(x)\gamma_\mu \psi^j(x)\,, \tag{66}$$

and then consider the difference

$$\Delta\Pi = \Pi^{ud}(0) - \Pi^{us}(0)\,. \tag{67}$$

The chiral computation of this quantity at order p^6 was first performed in [55] and the result was confirmed and expressed in terms of the canonical set of $O(p^6)$ LECs in [11]. The chiral expansion involves no LEC at all at chiral order p^4 and a single LEC at chiral order p^6, which is C_{61}^{r}. Using finite-energy sum rule techniques, the value of $\Delta\Pi$ can be determined from experiment [55] (earlier related calculations were performed in [56, 57]):

$$\Delta\Pi_{\mathrm{exp}} = 0.0203 \pm 0.0032\,. \tag{68}$$

This result translates into the following value for the $O(p^6)$ LEC:

$$C_{61}^{\mathrm{r}}(m_\rho) = (1.24 \pm 0.44) \times 10^{-3}\,\mathrm{GeV}^{-2}\,. \tag{69}$$

On the other hand, our resonance saturation model, using the results from (62) and the determination of the resonance parameters discussed above, yields

$$C_{61}^V = 2.10 \times 10^{-3}\,\mathrm{GeV}^{-2} \tag{70}$$

(using $F_V = F_\rho$; see (58)) which is in qualitative agreement with the experimental determination.

3.6 Comparison between resonance saturation and the dispersive representations

We can now return to the πK scattering amplitude and compute the vector meson contributions generated from the saturation of LECs C_i as shown above (62). We quote the result for the three subthreshold coefficients under consideration in this section,

$$\begin{aligned} C_{20}^+\big|_{C_i^V} &= \left[-\frac{7}{8}G_V^2 \frac{m_K^2 + m_\pi^2}{M_V^4} + \frac{3}{2}G_V^2 e_V^m \frac{m_K^2}{M_V^4} \right. \\ &\quad \left. - \frac{3}{\sqrt{2}} G_V \frac{2\,\hat{g}_{V1}^m m_K^2 + \hat{g}_{V2}^m m_\pi^2}{M_V^3}\right] \frac{m_\pi^4}{F_\pi^4}\,, \\ C_{01}^+\big|_{C_i^V} &= \left[2G_V^2 \frac{m_K^2 + m_\pi^2}{M_V^4} - 8G_V^2 e_V^m \frac{m_K^2}{M_V^4} \right. \\ &\quad \left. + 8\sqrt{2} G_V \frac{2\,\hat{g}_{V1}^m m_K^2 + \hat{g}_{V2}^m m_\pi^2}{M_V^3}\right] \frac{m_\pi^2 m_K^2}{F_\pi^4}\,, \\ C_{10}^-\big|_{C_i^V} &= \left[3G_V^2 \frac{m_K^2 + m_\pi^2}{M_V^4} - 4G_V^2 e_V^m \frac{m_K^2 + 2m_\pi^2}{M_V^4} \right. \\ &\quad \left. + 4\sqrt{2} G_V \frac{2(\hat{g}_{V1}^m + \hat{g}_{V2}^m) m_K^2 + (4\,\hat{g}_{V1}^m + \hat{g}_{V2}^m) m_\pi^2}{M_V^3}\right] \\ &\quad \times \frac{m_K m_\pi^3}{F_\pi^4}\,. \end{aligned} \tag{71}$$

A comparison of the numerical results for the resonance-saturated part of these subthreshold parameters between the vector field model and the antisymmetric tensor model is performed in Table 3. One can see that the differences are substantial. In two cases even the sign of the result is different.

One can perform a check of the resonance saturation model in the following way. Consider the set of subthreshold coefficients which can be written as sum rules with no subtractions. At this level, it is easy to identify a particular resonance R contribution: it suffices to restrict the integration region to the neighborhood of the resonance mass and to restrict the sum over partial waves to the one which corresponds to the spin of the resonance. This is illustrated in Fig. 2 which shows the integrands (in both the s and the t channel) associated with the coefficient C_{20}^+. In this case, the contribution from the vector

Table 3. Results on the $O(p^6)$ part involving the C_i^{r} LECs of some subthreshold coefficients, using two different vector resonance saturation models of these

	V model [15]	AT model
$C_{20}^+\big\|_{C_V^i}$	−0.005	−0.010
$C_{01}^+\big\|_{C_V^i}$	−0.27	0.30
$C_{10}^-\big\|_{C_V^i}$	−0.11	0.21

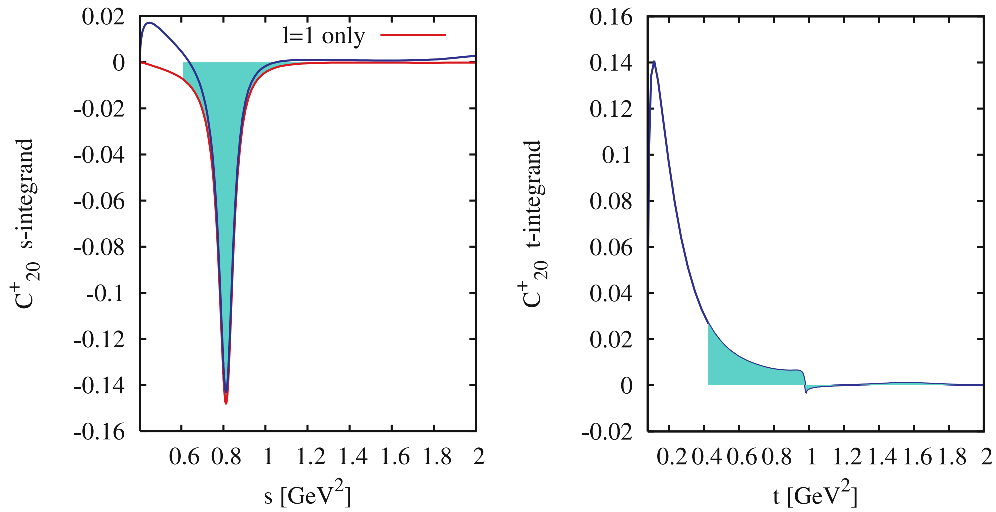

Fig. 2. C^+_{20} integrands: s channel (*left figure*) and t channel (*right figure*). The *shaded area* on the *left figure* isolates the $K^*(890)$ resonance contribution and on the *right figure* the scalar $f_0(980)$ one

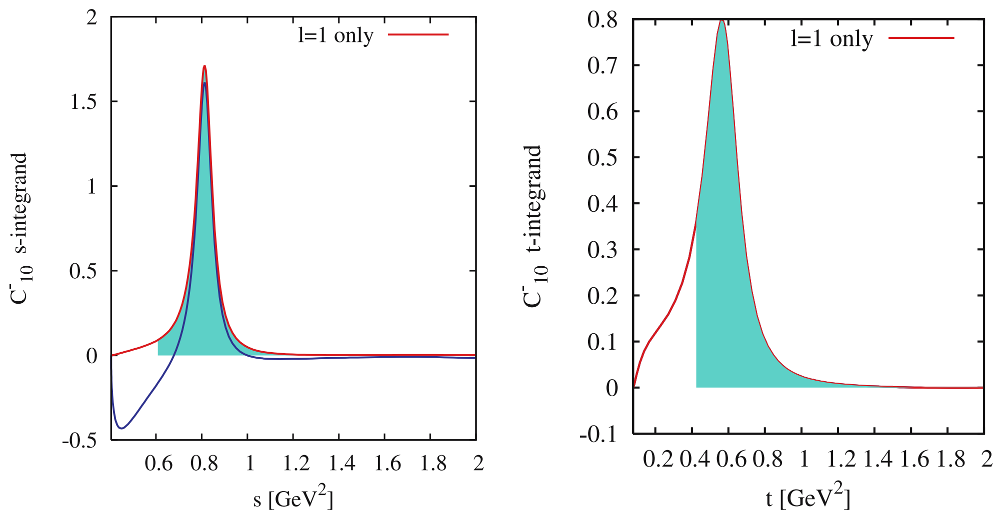

Fig. 3. C^-_{10} integrands: s channel (*left figure*) and t channel (*right figure*). The *shaded area* on the *left figure* isolates the $K^*(890)$ resonance contribution and on the *right figure* the $\rho(770)$ one

resonance can be isolated in the s channel, and the contributions from the scalar resonances can be identified in both the s and the t channel. Figure 3 illustrates the situation for the coefficient C^-_{10}: in this case the vector contribution appears in both the s and the t channel.

From the point of view of ChPT, now, we can split the contributions to a given subthreshold coefficient C_{ij} into one part, $C^{\rm loop}_{ij}$, which arises from loop diagrams, and one part, $C^{\rm tree}_{ij}$, which arises from tree level diagrams. The latter piece, up to chiral order p^6, involves terms linear in the $O(p^4)$ LECs $L^{\rm r}_i$, terms which are quadratic in the $O(p^4)$ LECs and, finally, those which are linear in the LECs $C^{\rm r}_i$. Both $C^{\rm loop}_{ij}$ and $C^{\rm tree}_{ij}$ depend on the chiral renormalization scale μ. Let us assume that a proper scale μ exists such that $C^{\rm loop}_{ij}$ corresponds to the low energy integration part of the coefficient C_{ij} and $C^{\rm tree}_{ij}$ to the higher energy part. We can then make a check of our resonance saturation model by computing $C^{\rm tree}_{ij}$ using the resonance-saturated values of the LECs L_i and C_i and comparing the result with the dispersive integral calculation in which the integral is computed over an energy range $E > E_0$. The lower boundary of the integration range should be somewhat below the resonance mass.

We will only consider the role of the vector mesons here. In the resonance saturation model, we correspondingly keep the terms proportional to the coupling G_V. The terms arising from the LECs $C^{\rm r}_i$ were shown in (71). Upon using the resonance model of [19] and retaining the contributions proportional to G_V the terms which are linear or quadratic in the LECs $L^{\rm r}_i$ yield

$$C^+_{20}\big|_{L+LL} = -\frac{3}{8}\frac{G_V^2}{M_V^2}\left[1-8\frac{c_d c_m\left(m_K^2-m_\pi^2\right)}{F_\pi^2 M_S^2}\right]\frac{m_\pi^4}{F_\pi^4}\,,$$
$$C^+_{01}\big|_{L+LL} = 2\frac{G_V^2}{M_V^2}\left[1-8\frac{c_d c_m\left(m_K^2-m_\pi^2\right)}{F_\pi^2 M_S^2}\right]\frac{m_K^2 m_\pi^2}{F_\pi^4}\,,$$
$$C^-_{10}\big|_{L+LL} = 3\frac{G_V^2}{M_V^2}\left[1-8\frac{c_d c_m\left(m_K^2-m_\pi^2\right)}{F_\pi^2 M_S^2}\right]\frac{m_K m_\pi^3}{F_\pi^4}\,. \quad (72)$$

The comparison, as discussed above, of the resonance saturation result with the dispersive resonance calculation is performed in Table 4. The table shows that the results from the antisymmetric tensor model for the relevant C_i when added to

Table 4. Comparison between vector resonance contributions to three subthreshold coefficients as computed from sum rules (last column) and as computed from resonance saturation models of the LECs. The second column shows the contributions which are linear and quadratic in the LECs L_i, while the third and fourth column show the additional effect of the LECs C_i using the vector or the antisymmetric tensor model respectively

	$L+LL$	$(L+LL+C)_V$	$(L+LL+C)_{AT}$	Sum rule
C^+_{20}	-0.0065	-0.012	-0.017	-0.017
C^+_{01}	0.439	0.17	0.74	0.66
C^-_{10}	0.185	0.08	0.40	0.40

the contributions linear and quadratic in the L_i compares rather well with the resonance contributions as computed from the sum rules.

3.7 A LEC combination with dominant vector contributions

In general, the low energy couplings get important contributions from the light vector mesons and also from the light scalar resonances [19]. Accounting for the scalar contributions is made difficult by several features. Firstly, the OZI rule is rather strongly violated in the scalar meson sector. This induces a large number of parameters in the resonance Lagrangian which cannot be determined unless some assumptions are made: see e.g. [34] for a recent discussion and some examples of such assumptions. A second difficulty is caused by the presence of the wide scalars (the σ or κ mesons). Interferences between the contributions from the wide scalars and the narrow ones lead to structures in the partial wave amplitudes (see e.g. Fig. 2 right) which are not well approximated by computing tree level diagrams from a resonance Lagrangian. For these reasons, it is useful to try to identify specific combinations of LECs which receive small contributions from the scalar mesons. We can generate one such combination by starting from πK subthreshold coefficients and forming the following combination:

$$C_{\rm NS} = C^+_{01} + \frac{2m_K}{m_\pi}\, C^-_{10}\,. \tag{73}$$

Indeed, this quantity satisfies an unsubtracted dispersion relation and, by construction, it receives no resonant S-wave contributions from either the t or the s channel. The only resonant contributions are from the $l \geq 1$ partial waves. The s channel integrand is shown in Fig. 4, while the t channel integrand is the same, up to a scale factor, as that shown in Fig. 3. Computing the integrals we find the experimental value of this quantity:

$$C_{\rm NS} = 4.27 \pm 0.17\,. \tag{74}$$

Using the chiral expansion for $C_{\rm NS}$ one finds that the following combination of $O(p^4)$ and $O(p^6)$ LECs is involved:

$$\begin{aligned} L_2^{\rm eff}(\mu) = L_2^{\rm r} + \left(m_K^2 + m_\pi^2\right)\left(-2C_4^{\rm r} + C_{10}^{\rm r} - 2C_{12}^{\rm r} + 2C_{22}^{\rm r}\right. \\ \left. +2C_{23}^{\rm r} - C_{25}^{\rm r}\right) + \left(4m_K^2 + 2m_\pi^2\right)\left(C_{11}^{\rm r} - 2C_{13}^{\rm r}\right)\,. \end{aligned} \tag{75}$$

According to the remarks made above, this combination of LECs receives no contributions from the scalar mesons corresponding to virtual exchanges in the πK scattering amplitude. It does, however, pick up contributions from the scalars via tadpole type diagrams[4]. Such contributions have been accounted for in our approach via flavor symmetry-breaking effects with the exception, however, of the LEC $C_{12}^{\rm r}$ (and for the $1/N_c$ suppressed LECs). This LEC receives no contribution from the resonance Lagrangian terms which we have considered. Fortunately, direct determinations exist for $C_{12}^{\rm r}$ based on the scalar form-factors with either $\Delta S = 0$ [14] or $\Delta S = 1$ [28]. The latter determination seems more precise and gives a value in the range $-0.6 \leq 10^4\, C_{12}^{\rm r}(m_\rho) \leq 0.6\ {\rm GeV}^{-2}$, which implies that the corresponding contribution in (75) is negligibly small.

We can determine the experimental value of $L^{\rm eff}$ from the experimental value of the combination $C_{\rm NS}$ (74) and its chiral expansion. If we use the expansion up to order p^4 we find

$$L_2^{\rm eff}(m_\rho)\big|_{p^4} \simeq 1.32 \times 10^{-3}\,. \tag{76}$$

This value agrees rather well with that found from the resonance saturation model $L_2^V = 1.2 \times 10^{-3}$ [19]. If we now include the $O(p^6)$ correction in the chiral expansion we find

$$L_2^{\rm eff}(m_\rho)\big|_{p^4+p^6} = (0.16 \pm 0.08) \times 10^{-3}\,. \tag{77}$$

We would like now to compare with the result from the resonance saturation model also including $O(p^6)$ corrections which has the following expression:

$$\begin{aligned} L_2^{\rm eff}\big|_V = \frac{G_V^2}{4M_V^2}\left\{1 + \frac{m_K^2 + m_\pi^2}{M_V^2}\left[1 - 2e_V^m\right.\right. \\ \left.\left. +2\sqrt{2}\frac{M_V}{G_V}(2\,\hat{g}_{V1}^m + \hat{g}_{V2}^m)\right]\right\}\,. \end{aligned} \tag{78}$$

Numerically, using the results from Sect. 3.4, one obtains

$$L_2^{\rm eff}\big|_V \simeq 2.04 \times 10^{-3}\,. \tag{79}$$

Keeping in mind that the determination of the resonance Lagrangian couplings is approximate (due, in particular, to the use of large N_c type approximations), it is nevertheless clear that the value of $L_2^{\rm eff}$ obtained above (79) from our resonance saturation model differs quite substantially (by about a factor of ten) from the experimental determination of $L_2^{\rm eff}(\mu)$ when $\mu = m_\rho$.

3.8 Discussion

This problem cannot be attributed to the resonance model itself since we have checked that the results do correspond, at least approximately, to the contribution from the resonance region in the sum rule expression of $C_{\rm NS}$ (the integrand is shown in Fig. 4). It must therefore be concluded that the values of the LECs can fail to be dominated by the resonance contributions at $O(p^6)$ with $\mu = m_\rho$.

One obvious possible reason for the failure of resonance saturation is that the variation of the LECs as a function of μ

[4] We thank Roland Kaiser for pointing this out to us.

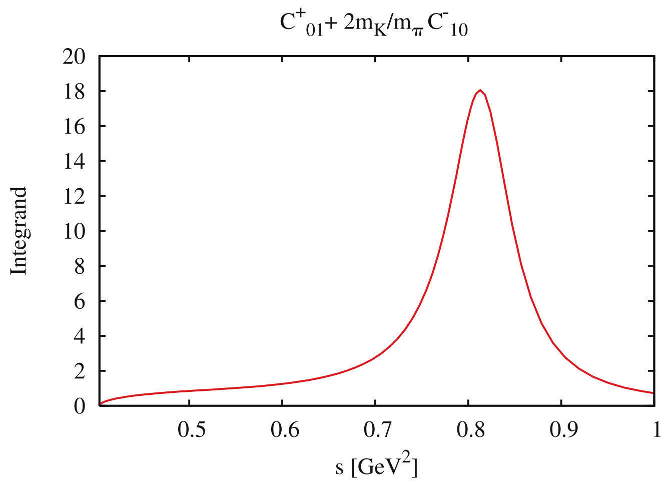

Fig. 4. Integrand in the s variable for the subthreshold coefficient $C_{\rm NS}$ defined in (73)

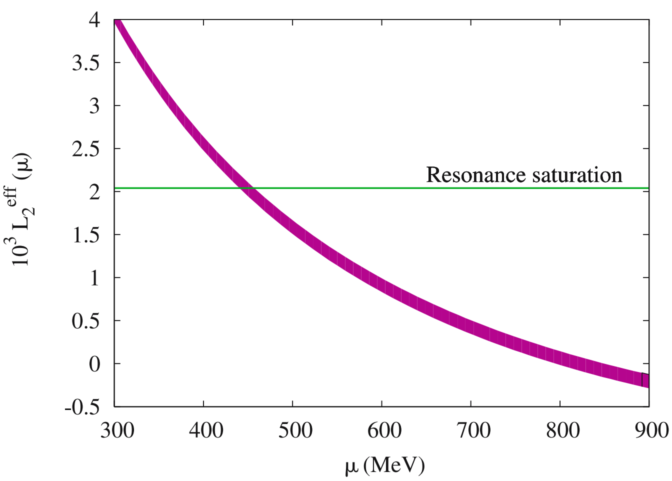

Fig. 5. Chiral coupling combination $L_2^{\rm eff}(\mu)$ (see (75)) as a function of the scale μ compared with the vector meson saturation result

at $O(p^6)$ can be much faster than it is for the $O(p^4)$ LECs. This is illustrated in Fig. 5 which shows the behaviour of $L_2^{\rm eff}$ as a function of the scale. The figure shows that, in fact, a scale μ_0 does exist such that resonance saturation of $L_2^{\rm eff}$ is exact, but its value, $\mu_0 \simeq 0.45$ GeV is significantly smaller than m_ρ.

One must also keep in mind that the renormalized coupling constants are obtained from the bare ones by a minimal subtraction procedure. Their values thus depend both on the regularization scheme and on the subtraction convention. The procedure adopted in ChPT (based on dimensional regularization and modified minimal subtraction) was shown to lead to natural values for the couplings at order p^4. This, however, is not guaranteed to remain true at arbitrary higher orders.

A remark is in order, finally, concerning the chiral expansion of the quantity $L_2^{\rm eff}$. In the resonance saturation model, the contribution of order p^6 is rather large, amounting to a 50% correction as compared to the $O(p^4)$ one. At first sight, the situation seems to be worse for $L_2^{\rm eff}(\mu)$: if we set $\mu = m_\rho$, the contribution of order p^6 practically cancels that of order p^4. In this case, however, the relative contributions strongly depend on the scale: if we take $\mu \simeq 0.55$ GeV the $O(p^6)$ contribution will be much smaller than the $O(p^4)$ one, while if we take $\mu \simeq 0.45$ GeV the relative contributions become similar to those in the resonance saturation model.

4 Summary

Our goal was to extract some model independent information about the $O(p^6)$ chiral coupling constants, about which little is known at present, and probe the validity, in this sector, of the idea of resonance dominance. We used input from the πK scattering amplitude in the subthreshold region derived from experimental data using dispersion relations. In this way, we generated three constraints on the four LECs C_1 to C_4 and three constraints on eleven LECs among C_5 to C_{22}. These are associated with chiral operators which involve one insertion of the quark mass matrix. In line with the earlier work of [15] it appears natural, assuming resonance dominance, to associate the values of these LECs with flavor symmetry breaking in the light resonance sector. In order to implement this, we have considered a (vector) resonance Lagrangian which is more general than the one used in [15]. We determined all the coupling constants in this Lagrangian from experiment, in the spirit of a large N_c approach. In principle, a more consistent approach to the determination of such couplings is to appeal to asymptotic constraints [42]. In practice, the two approaches usually give similar results and, furthermore, it is often not possible to satisfy all the relevant asymptotic constraints using a minimal number of resonances (e.g. [49, 58]). Here, in order to test some of our estimates for the resonance content of the LECs, we have used unsubtracted sum rules in which one restricts the integration range to the resonance region.

One of our initial motivations was to try to understand the reason for a number of significant discrepancies between the chiral $O(p^6)$ predictions of [15] for certain subthreshold expansion parameters of the πK amplitude and the dispersive results. We found that improving the vector resonance Lagrangian does not help in resolving these discrepancies. We made no attempt to improve the scalar resonance Lagrangian, but we identified a specific combination of chiral LECs which should be insensitive to that sector (beyond the effect of generating flavor symmetry breaking). A clear outcome of our analysis is that, if one sets the value of the chiral scale μ equal to the ρ meson mass, then the value of this combination of LECs is not dominated by the resonance contribution. We have also encountered examples for which resonance dominance was reasonable; see Sect. 3.5.1. This suggests that in parallel to the efforts which are pursued in order to develop consistent resonance models (e.g. [33]) one should also try to obtain further direct determinations of the LECs C_i.

This result may be compared with the observation made in the baryon sector of ChPT [59] already at one loop. In dimensional regularization, the one-loop corrections to the baryon masses were found to be rather large, requiring, in order to compensate for that, that the low energy couplings be set to values which are unnaturally large. The origin of the problem was traced to the regularization procedure and the physical interpretation of the chiral scale μ. One expects μ to correspond approximately to a momentum cutoff in the loop integrals. The authors of [59] show that this expectation can break down when unequal mass particles propagate inside the loops. As a possible cure to this problem they proposed to use a regularization method different from dimensional regularization.

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